

Rates and Asymmetries of $B \rightarrow \pi \ell^+ \ell^-$ Decays

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The LHCb experiment observed $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay with 1.0 fb^{-1} data, which is the first measurement of a flavor changing neutral current $b \rightarrow d \ell^+ \ell^-$ decay ($\ell = e, \mu$). Based on QCD factorization, we give Standard Model predictions for the branching ratios, direct CP asymmetries, and isospin asymmetry for $B \rightarrow \pi \ell^+ \ell^-$ decays, in the kinematic region where the dilepton invariant mass is small. We find that the contribution from weak annihilation enhances the direct CP asymmetry for low $\ell^+ \ell^-$ pair mass. Anticipating improved measurements, we assess the utility of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ observables, when combined with $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow K^+ \ell^+ \ell^-$, for determining CKM parameters in the future.

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I. INTRODUCTION

After much anticipation for possible new physics in $b \rightarrow s$ transitions, all measurements at the LHC with 2011–2012 data turned out consistent with Standard Model (SM) expectations, and further progress would take more data to unfold. The focus, and perhaps sensitivity to New Physics, may now be on $b \rightarrow d$ rare decays.

The LHCb has observed [1] the $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay with measured branching ratio

$$\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = (2.3 \pm 0.6 \text{ (stat.)} \pm 0.1 \text{ (syst.)}) \times 10^{-8}, \quad (1)$$

at 5.2σ significance. This is the first observation of a flavor changing neutral current (FCNC) $b \rightarrow d \ell^+ \ell^-$ transition, as the B factories were only able to set limits [2, 3]. We summarize relevant data in Table I. As data accumulates, the era of FCNC $b \rightarrow d$ rare decays has dawned. The Belle II experiment under construction should be able to measure $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ decay in the future.

Previous theoretical studies are based on naive factorization [4–7] or the perturbative QCD approach [8]. In this paper, we provide a first estimate for the exclusive $B \rightarrow \pi \ell^+ \ell^-$ ($\ell = e$ or μ) decays based on the QCD factorization (QCDF) framework [9–15]. This framework has been introduced for exclusive nonleptonic B -meson decays [9, 10], and applied for exclusive radiative and semileptonic B -meson FCNC decays [11–15], which include the well-studied $B \rightarrow K^* \ell^+ \ell^-$. We extend the known result for the $B \rightarrow \rho \ell^+ \ell^-$ decays [15] to the $B \rightarrow \pi \ell^+ \ell^-$ case. We provide the SM predictions for the decay rates within QCDF, as well as associated CP asymmetries and isospin asymmetry at $\mathcal{O}(\alpha_s)$.

In contrast to naive factorization treatments, one advantage of the QCDF framework is that one can include hard-spectator-scattering effects, such as weak annihilation diagrams. This is crucial for addressing CP violation (CPV) and isospin asymmetry in these decays, which are of great interest even within SM because $V_{ud}^* V_{ub}$ carries CPV phase. In contrast, such effects are suppressed by $V_{us}^* V_{ub}$ for $b \rightarrow s \ell^+ \ell^-$ processes. The $B \rightarrow \pi \ell^+ \ell^-$ modes may also be used to probe New Physics, such as two

Higgs doublet model [4, 5], R-parity violating supersymmetry [6], and fourth generation [16].

This paper is a followup of the last paper, where some results were used in conjunction with the study of $B_d \rightarrow \mu^+ \mu^-$ mode. In this paper we focus on SM expectations for $B \rightarrow \pi \ell^+ \ell^-$. After presenting the basic formulas and input parameters in Sec. II, the SM predictions are given in Sec. III. We discuss prospects for constraining CKM parameters in Sec. IV, before giving our conclusion. Some details are placed in Appendices.

II. BASIC FORMULAS AND INPUT

The effective Hamiltonian for $b \rightarrow d \ell^+ \ell^-$ is given by

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \left[\lambda_t \mathcal{H}_{\text{eff}}^{(t)} + \lambda_u \mathcal{H}_{\text{eff}}^{(u)} \right] + \text{h.c.}, \quad (2)$$

in SM, where $\lambda_q = V_{qd}^* V_{qb}$ ($q = u, t$) and

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{(u)} &\equiv C_1 (\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2 (\mathcal{O}_2^c - \mathcal{O}_2^u), \\ \mathcal{H}_{\text{eff}}^{(t)} &\equiv C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{10} C_i \mathcal{O}_i. \end{aligned} \quad (3)$$

TABLE I. A summary of experimental results for the $B \rightarrow \pi \ell^+ \ell^-$ decay branching ratios (in unit of 10^{-8}). The values quoted for Belle (based on 657M $B\bar{B}$ pairs) and BaBar (based on 471M $B\bar{B}$ pairs) are the 90% C.L. upper limits. The less stringent limit by Belle on $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ has more to do with a large central value (for both $\mu^+ \mu^-$ and $e^+ e^-$), than a somewhat larger error than BaBar.

Mode	LHCb [1]	Belle [2]	BaBar [3]
$B^+ \rightarrow \pi^+ \mu^+ \mu^-$	$2.3 \pm 0.6 \pm 0.1$	< 6.9	< 5.5
$B^+ \rightarrow \pi^+ e^+ e^-$	–	< 8.0	< 12.5
$B^+ \rightarrow \pi^+ \ell^+ \ell^-$	–	< 4.9	< 6.6
$B^0 \rightarrow \pi^0 \mu^+ \mu^-$	–	< 18.4	< 6.9
$B^0 \rightarrow \pi^0 e^+ e^-$	–	< 22.7	< 8.4
$B^0 \rightarrow \pi^0 \ell^+ \ell^-$	–	< 15.4	< 5.3

We follow the operator basis of Ref. [15], i.e.

$$\begin{aligned}\mathcal{O}_7 &= -\frac{e\hat{m}_b}{8\pi^2}\bar{d}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}, \\ \mathcal{O}_8 &= -\frac{g_s\hat{m}_b}{8\pi^2}\bar{d}_i\sigma^{\mu\nu}(1+\gamma_5)T_{ij}^A b_j G_{\mu\nu}^A, \\ \mathcal{O}_9 &= \frac{\alpha}{2\pi}[\bar{d}\gamma^\mu(1-\gamma_5)b][\bar{\ell}\gamma_\mu\ell], \\ \mathcal{O}_{10} &= \frac{\alpha}{2\pi}[\bar{d}\gamma^\mu(1-\gamma_5)b][\bar{\ell}\gamma_\mu\gamma_5\ell],\end{aligned}\quad (4)$$

where repeated indices are summed, $\alpha = e^2/4\pi$ and $\hat{m}_b(\mu)$ denotes the b quark mass in the $\overline{\text{MS}}$ scheme. We have eliminated $\lambda_c = V_{cd}^*V_{cb}$ in Eq. (2) by using the unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$.

The effective Hamiltonian for $b \rightarrow s\ell^+\ell^-$ can be obtained by replacing d by s in the equations above. In this case, the $\lambda_u\mathcal{H}_{\text{eff}}^{(u)}$ term can be neglected due to smallness of $|V_{us}^*V_{ub}| \ll |V_{ts}^*V_{tb}|$. For $b \rightarrow s\ell^+\ell^-$, therefore, the effective Hamiltonian can be factorized by the single CKM factor $V_{ts}^*V_{tb}$ to good approximation. In contrast, λ_u and λ_t are comparable in magnitude for $b \rightarrow d\ell^+\ell^-$, with the sizable phase difference,

$$\phi_2 \equiv \alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad (5)$$

for which global analyses give $\phi_2 \sim 89^\circ$ [17]. As λ_u enters the amplitudes with tree-level operators $\mathcal{O}_2^q = [\bar{d}\gamma^\mu(1-\gamma_5)q][\bar{q}\gamma_\mu(1-\gamma_5)b]$ ($q = u, c$), its effects can be numerically large. Hence, the amplitudes for $b \rightarrow d\ell^+\ell^-$ have more complex structure than the $b \rightarrow s\ell^+\ell^-$ amplitudes, resulting in richer phenomenology such as CPV, which will be discussed in this article. The following formulas can also be applied to exclusive $B \rightarrow K\ell^+\ell^-$ decays in a straightforward manner, keeping the above remarks in mind.

We use next-to-next-to-leading logarithmic (NNLL) results for $C_{9,10}$ and next-to-leading logarithmic (NLL) results for C_{1-6} and $C_{7,8}^{\text{eff}}$, which are necessary to calculate $b \rightarrow d\ell^+\ell^-$ at $\mathcal{O}(\alpha_s)$. These were obtained by using the formal solutions of the renormalization group equations given in Ref. [12], with two-loop matching conditions [18] and the three-loop anomalous dimension matrix [19–21], as well as updated input parameters.

In calculating the amplitude for the exclusive decay $\bar{B} \rightarrow P\ell^+\ell^-$, where P denotes light pseudoscalar mesons π^- , π^0 , K^- and \bar{K}^0 , one has to deal with matrix elements for the various operators appearing in Eq. (2). The matrix elements for $\mathcal{O}_{9,10}$ are simply expressed by the form factors, as they are bilinear in the quark fields. The contributions from remaining operators enter through the process with virtual photon γ^* , namely, $\bar{B} \rightarrow P\gamma^* \rightarrow P\ell^+\ell^-$, which can be parameterized as

$$\begin{aligned}c_P \langle \gamma^*(q, \mu) P(p') | \mathcal{H}_{\text{eff}}^{(i)} | \bar{B}(p) \rangle \\ = -\frac{em_b}{4\pi^2} \frac{\mathcal{T}_P^{(i)}(q^2)}{M_B} [q^2(p^\mu + p'^\mu) - (M_B^2 - m_P^2)q^\mu],\end{aligned}\quad (6)$$

for $i = t, u$, where $|P\rangle$ denotes $|\pi^- \rangle$ or $|K^- \rangle$ ($|\pi^0 \rangle$ or $|\bar{K}^0 \rangle$) for B^- (\bar{B}^0) decay. The isospin factor c_P is $-\sqrt{2}$ for $P = \pi^0$, and 1 for $P = \pi^-, K^-, \bar{K}^0$ which differs in normalization from Ref. [15].

In the QCDF framework, the $\bar{B} \rightarrow P\gamma^*$ amplitude can be decomposed as [12, 15]

$$\begin{aligned}\mathcal{T}_P^{(i)} &= \xi_P C_P^{(i)} \\ &+ \frac{\pi^2 f_B f_P}{N_c M_B} \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \int_0^1 du \phi_P(u) T_{P,\pm}^{(i)}(u, \omega),\end{aligned}\quad (7)$$

where $C_P^{(i)}$ and $T_{P,\pm}^{(i)}$ are described by short-distance physics, their perturbative expressions are given in Appendix A. Information from long-distance physics is encoded in the $\bar{B} \rightarrow P$ form factor ξ_P , and the light-cone distribution amplitudes, $\Phi_{B,\pm}$ for the B meson and ϕ_P for the light pseudoscalar meson.

Eq. (7) relies on the heavy quark limit of the b quark. It further assumes the energy of the final state meson P in the B -meson rest frame, denoted as $E_P = (M_B^2 + m_P^2 - q^2)/2M_B$, is large enough, i.e. $E_P \simeq M_B/2$ or $q^2 \ll M_B^2$. The QCDF approach is, hence, restricted in the kinematical region where the invariant mass of the lepton pair q^2 is small. Besides this, the $\bar{B} \rightarrow P\gamma^*$ amplitude suffers from nonperturbative corrections due to the near-threshold $u\bar{u}$ and $c\bar{c}$ intermediate states, which form the ρ, ω, \dots and charmonium resonances. For better theoretical control, we limit our analysis in the $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ region where the QCDF approach is expected to work, while the resonance contaminations would also be avoided.¹ This is more conservative compared to the $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ range used in Ref. [15], or $1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2$ in Ref. [7]. Comparison will be made for these regions.

Including the contributions from $\mathcal{O}_{9,10}$, the amplitude for $\bar{B} \rightarrow P\ell^+\ell^-$ is given by

$$\begin{aligned}\mathcal{M}(\bar{B} \rightarrow P\ell^+\ell^-) \\ = \frac{G_F \alpha}{2\sqrt{2}\pi} c_P^{-1} \xi_P \left[\left(\lambda_t C_{9,P}^{(t)} + \lambda_u C_{9,P}^{(u)} \right) (p^\mu + p'^\mu) (\bar{\ell}\gamma_\mu\ell) \right. \\ \left. + \lambda_t C_{10}(p^\mu + p'^\mu) (\bar{\ell}\gamma_\mu\gamma_5\ell) \right],\end{aligned}\quad (8)$$

where

$$\begin{aligned}C_{9,P}^{(t)}(q^2) &= C_9 + \frac{2m_b}{M_B} \frac{\mathcal{T}_P^{(t)}(q^2)}{\xi_P(q^2)}, \\ C_{9,P}^{(u)}(q^2) &= \frac{2m_b}{M_B} \frac{\mathcal{T}_P^{(u)}(q^2)}{\xi_P(q^2)}.\end{aligned}\quad (9)$$

¹ The resonance effects on the $B \rightarrow \pi\ell^+\ell^-$ decays were studied based on naive factorization in Ref. [4]. Their result on the dilepton invariant mass distribution of the branching ratio and of the CP asymmetry supports safety of this q^2 range. The recent study [22] in relativistic quark model, however, seems to suggest that resonance effects may still be important, even for our conservative q^2 range.

In the above amplitude, the term proportional to m_ℓ is neglected. Therefore, hereafter we do not distinguish the cases of $\ell = e$ and μ .

The kinematics of $\bar{B} \rightarrow P\ell^+\ell^-$ decay can be described by the dilepton invariant mass q^2 and $\cos\theta$, where θ is the angle between the momentum of ℓ^+ and the momentum of P in the rest frame of the lepton pair. The $\cos\theta$ dependence of the decay might be interesting in relation to the forward-backward asymmetry. However, it is zero in the current case as the double differential decay rate behaves as $d^2\Gamma/dq^2 d(\cos\theta) \propto (1 - \cos^2\theta)$. Therefore, we discuss only q^2 dependence.

The differential branching ratio is given by

$$\begin{aligned} & \frac{d\mathcal{B}}{dq^2}(\bar{B} \rightarrow P\ell^+\ell^-) \\ &= S_{P\tau B} \frac{G_F^2 M_B^3}{96\pi^3} \left(\frac{\alpha}{4\pi}\right)^2 \lambda(q^2, m_P^2)^3 \xi_P(q^2)^2 \\ & \quad \times \left(\left| \lambda_t C_{9,P}^{(t)}(q^2) + \lambda_u C_{9,P}^{(u)}(q^2) \right|^2 + |\lambda_t|^2 C_{10}^2 \right) \\ &= S_{P\tau B} \frac{G_F^2 M_B^3}{96\pi^3} \left(\frac{\alpha}{4\pi}\right)^2 \lambda(q^2, m_P^2)^3 \xi_P(q^2)^2 |\lambda_t|^2 \\ & \quad \times \left(\left| C_{9,P}^{(t)}(q^2) - R_{ut} e^{i\phi_2} C_{9,P}^{(u)}(q^2) \right|^2 + C_{10}^2 \right), \quad (10) \end{aligned}$$

where

$$\begin{aligned} & \lambda(q^2, m_P^2) \\ &= \left[\left(1 - \frac{q^2}{M_B^2}\right)^2 - \frac{2m_P^2}{M_B^2} \left(1 + \frac{q^2}{M_B^2}\right) + \frac{m_P^4}{M_B^4} \right]^{\frac{1}{2}}. \quad (11) \end{aligned}$$

$S_P = 1/c_P^2 = 1/2$ for $P = \pi^0$, and $S_P = 1$ for $P = \pi^-, K^-, \bar{K}^0$, and we have defined

$$\lambda_u/\lambda_t \equiv -R_{ut} e^{i\phi_2} \quad (12)$$

for $\bar{B} \rightarrow \pi\ell^+\ell^-$ in SM.

The branching ratio for the CP-conjugate mode is obtained by flipping the sign of the weak phase in Eq. (10), i.e., by replacing $\phi_2 \rightarrow -\phi_2$. We then define the q^2 -dependent direct CP asymmetries as²

$$\begin{aligned} & A_{\text{CP}}^+(q^2) \\ &\equiv \frac{d\mathcal{B}(B^- \rightarrow \pi^-\ell\ell)/dq^2 - d\mathcal{B}(B^+ \rightarrow \pi^+\ell\ell)/dq^2}{d\mathcal{B}(B^- \rightarrow \pi^-\ell\ell)/dq^2 + d\mathcal{B}(B^+ \rightarrow \pi^+\ell\ell)/dq^2}, \\ & A_{\text{CP}}^0(q^2) \\ &\equiv \frac{d\mathcal{B}(\bar{B}^0 \rightarrow \pi^0\ell\ell)/dq^2 - d\mathcal{B}(B^0 \rightarrow \pi^0\ell\ell)/dq^2}{d\mathcal{B}(\bar{B}^0 \rightarrow \pi^0\ell\ell)/dq^2 + d\mathcal{B}(B^0 \rightarrow \pi^0\ell\ell)/dq^2}. \quad (13) \end{aligned}$$

We also define the q^2 -dependent isospin asymmetry as³

$$A_I(q^2) \equiv \frac{\tau_{B^0}}{2\tau_{B^\pm}} \frac{d\bar{\mathcal{B}}(B^+ \rightarrow \pi^+\ell\ell)/dq^2}{d\bar{\mathcal{B}}(B^0 \rightarrow \pi^0\ell\ell)/dq^2} - 1, \quad (14)$$

where $\bar{\mathcal{B}}$ stand for taking the CP-average.

By integrating over the numerator and the denominator in Eq. (13) separately, we further define the q^2 -averaged direct CP asymmetries as

$$\begin{aligned} & \langle A_{\text{CP}}^+ \rangle \\ &\equiv \frac{\mathcal{B}(B^- \rightarrow \pi^-\ell\ell) - \mathcal{B}(B^+ \rightarrow \pi^+\ell\ell)}{\mathcal{B}(B^- \rightarrow \pi^-\ell\ell) + \mathcal{B}(B^+ \rightarrow \pi^+\ell\ell)}, \\ & \langle A_{\text{CP}}^0 \rangle \\ &\equiv \frac{\mathcal{B}(\bar{B}^0 \rightarrow \pi^0\ell\ell) - \mathcal{B}(B^0 \rightarrow \pi^0\ell\ell)}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^0\ell\ell) + \mathcal{B}(B^0 \rightarrow \pi^0\ell\ell)}, \quad (15) \end{aligned}$$

where the branching ratios \mathcal{B} are obtained by integrating Eq. (10) over a certain q^2 range. Similarly, we define the q^2 -averaged isospin asymmetry as,

$$\langle A_I \rangle \equiv \frac{\tau_{B^0}}{2\tau_{B^\pm}} \frac{\bar{\mathcal{B}}(B^+ \rightarrow \pi^+\ell\ell)}{\bar{\mathcal{B}}(B^0 \rightarrow \pi^0\ell\ell)} - 1. \quad (16)$$

With input parameters as summarized in Table II, the numerical values for the Wilson coefficients at the scale $\mu = m_b$ are given in Table III. Let us briefly explain some of the input parameters in Table II.

There are three form factors relevant to $\bar{B} \rightarrow P\ell^+\ell^-$ in SM, which are usually denoted as $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$. In the heavy quark and large recoil energy (or small q^2) limit, it is known [11, 37] that the three form factors are related by symmetry, and are described by the single soft form factor $\xi_P(q^2)$. The symmetry relations are broken by $\mathcal{O}(\alpha_s)$ corrections, but one may choose the factorization scheme where $f_+(q^2) = \xi_P(q^2)$ holds to all orders in perturbation theory. The $\mathcal{O}(\alpha_s)$ corrections to other two form factors can be estimated perturbatively [11], leading to the factorizable corrections, $C_P^{(f,t)}$ and $T_{P,+}^{(f,t)}$, given in Eq. (A6) of Appendix A.

For the $\bar{B} \rightarrow \pi$ form factor $\xi_\pi(q^2)$, we adopt the above factorization scheme with the following fit formula of Ref. [30], obtained by QCD light-cone sum rule with aid of the measured q^2 -distribution of $B \rightarrow \pi\ell\nu$,

$$\xi_\pi(q^2) = \frac{\xi_\pi(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{\text{BK}} q^2/m_B^2)}. \quad (17)$$

The numerical values for the normalization $\xi_\pi(0)$ and the slope parameter α_{BK} are given in Table II.

² Our definitions of the CP asymmetries in Eqs. (13) and (15) are similar to the corresponding ones for $B^+ \rightarrow K^+\mu^+\mu^-$ [23] and $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [24] studied by the LHCb experiment.

³ Our definition of the isospin asymmetry in Eqs. (14) and (16) is similar to the one used by Belle [25] and BaBar [26], as well as the theoretical paper [15] for $B \rightarrow \rho\gamma$. We differ from the LHCb convention [27] used for $B \rightarrow K^{(*)}\mu^+\mu^-$.

TABLE II. Summary of input parameters, taken from the Particle Data Group [17] unless otherwise stated. We use three-loop running for QCD coupling α_s with the listed initial value $\alpha_s(M_Z)$. Our treatment of m_b follows Ref. [12, 15] by choosing the potential-subtracted (PS) mass [28] as input. We define $\mathcal{B}_{\pi\ell\nu}^{\text{exp}} = \mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu)_{q^2 < 12 \text{ GeV}^2}^{\text{exp}}$ as described in the text. The errors explicitly shown are taken into account in our error analysis.

α	1/137	λ	0.22535 ± 0.00065
$\sin^2 \theta_W$	0.23	A	$0.811^{+0.022}_{-0.012}$
G_F	$1.166 \times 10^{-5} \text{ GeV}^{-2}$	$\bar{\rho}$	$0.131^{+0.026}_{-0.013}$
$\alpha_s(M_Z)$	0.1184 ± 0.0007	$\bar{\eta}$	$0.345^{+0.013}_{-0.014}$
M_W	80.4 GeV	$\xi_\pi(0)$	$0.26^{+0.04}_{-0.03} [30]$
M_Z	91.2 GeV	α_{BK}	$0.53 \pm 0.06 [30]$
$m_{t,\text{pole}}$	173.5 GeV	a_2^π	$0.25 \pm 0.15 [31]$
$m_{b,\text{PS}}(2 \text{ GeV})$	$(4.6 \pm 0.1) \text{ GeV} [28]$	a_4^π	$-a_2^\pi + (0.1 \pm 0.1) [32]$
$m_{c,\text{pole}}$	1.67 GeV	$\xi_K(0)$	$0.36^{+0.05}_{-0.04} [33]$
f_π	$(130.41 \pm 0.03 \pm 0.20) \text{ MeV}$	a_1^K	$0.10 \pm 0.04 [34]$
f_K	$(156.1 \pm 0.2 \pm 0.8 \pm 0.2) \text{ MeV}$	a_2^K	$0.25 \pm 0.15 [31, 33]$
f_B	$(190.6 \pm 4.7) \text{ MeV} [29]$	$\xi_K(0)/\xi_\pi(0)$	$1.38^{+0.11}_{-0.10} [33]$
τ_{B^0}	$1.52 \times 10^{-12} \text{ s}$	$\lambda_{B,+}(1.5 \text{ GeV})$	$(0.485 \pm 0.115) \text{ GeV} [15, 35]$
τ_{B^\pm}	$1.64 \times 10^{-12} \text{ s}$	$\mathcal{B}_{\pi\ell\nu}^{\text{exp}}$	$(0.81 \pm 0.02 \pm 0.03) \times 10^{-4} [36]$

TABLE III. The SM Wilson coefficients at the scale $\mu = 4.6 \text{ GeV}$ in leading logarithmic (LL), next-to-leading logarithmic (NLL) and next-to-next-to-leading logarithmic order (NNLL). Input parameters listed in Table II are used.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7^{eff}	C_8^{eff}	C_9	C_{10}
LL	-0.5093	1.0256	-0.0050	-0.0686	0.0005	0.0010	-0.3189	-0.1505	2.0111	0
NLL	-0.3001	1.0080	-0.0047	-0.0827	0.0003	0.0009	-0.2969	-0.1642	4.1869	-4.3973
NNLL	-	-	-	-	-	-	-	-	4.2607	-4.2453

The light-cone distribution amplitude ϕ_π is given by

$$\phi_\pi(u) = 6u(1-u) \left[1 + a_2^\pi C_2^{(3/2)}(2u-1) + a_4^\pi C_4^{(3/2)}(2u-1) + \dots \right], \quad (18)$$

where $C_n^{(3/2)}(x)$ are Gegenbauer polynomials. Following Ref. [30], we adopt the Gegenbauer coefficients a_2^π [31] and the combination $a_2^\pi + a_4^\pi$ [32] listed in Table II, by neglecting $a_{n>4}^\pi$ and scale dependence.

For B meson light-cone distribution amplitudes, we follow Ref. [12] and adopt simple model functions [38],

$$\Phi_{B,+}(\omega) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}, \quad \Phi_{B,-}(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0}. \quad (19)$$

These enter in the decay amplitude through the moments $\lambda_{B,+}^{-1}$ and $\lambda_{B,-}^{-1}(q^2)$, for which the model gives

$$\lambda_{B,+}^{-1} = \int_0^\infty d\omega \frac{\Phi_{B,+}(\omega)}{\omega} = \omega_0^{-1}, \quad (20)$$

$$\begin{aligned} \lambda_{B,-}^{-1}(q^2) &= \int_0^\infty d\omega \frac{\Phi_{B,-}(\omega)}{\omega - q^2/M_B - i\epsilon} \\ &= \frac{e^{-q^2/(M_B\omega_0)}}{\omega_0} [-\text{Ei}(q^2/M_B\omega_0) + i\pi], \end{aligned} \quad (21)$$

where $\text{Ei}(z)$ is the exponential integral function. We choose $\lambda_{B,+}$ as input, with numerical value obtained by QCD sum rule calculation [15, 35]. $\lambda_{B,-}^{-1}$ appears via the weak annihilation term, and the imaginary part in Eq. (21) serves as a crucial *source of strong phase* which is necessary for CPV. The model functions introduce theoretical uncertainty, which is discussed in Appendix B.

III. SM PREDICTIONS

With the input parameters in Table II and the SM Wilson coefficients in Table III, we give predictions for the $B \rightarrow \pi \ell^+ \ell^-$ decays. In Table IV, the numerical values of the amplitudes $\mathcal{C}_{9,P}^{(t,u)}$ and their breakdowns at $q^2 = 2 \text{ GeV}^2$ and 5 GeV^2 are shown. We note that $\mathcal{C}_{9,P}^{(t)}$ is dominated by C_9 , which arises from the electroweak penguin and W -box diagrams. For $B^0 \rightarrow \pi^0 \ell^+ \ell^-$, the magnitude of $\mathcal{C}_{9,P}^{(u)}$ is minor compared to $\mathcal{C}_{9,P}^{(t)}$. On the other hand, for $B^+ \rightarrow \pi^+ \ell^+ \ell^-$, the magnitude of $\mathcal{C}_{9,P}^{(u)}$ becomes comparable to $\mathcal{C}_{9,P}^{(t)}$ at $q^2 = 2 \text{ GeV}^2$. This is mainly due to the aforementioned large imaginary part coming from the $\mathcal{O}(\alpha_s^0)$ hard-spectator-scattering term $T^{(0)}$, corresponding to the term that includes $\hat{T}_{P,-}^{(0,u)}$ in

TABLE IV. Numerical values and breakdowns for the amplitudes $C_{9,P}^{(t,u)}$ at $q^2 = 2 \text{ GeV}^2$ and 5 GeV^2 , for $B \rightarrow \pi \ell \ell$ (Table 5 in Ref. [15] gives analogous values for $B \rightarrow \rho \ell \ell$ at $q^2 = 5 \text{ GeV}^2$). Each term is classified into two categories: (1) form factor term, which includes C_9 , $Y(q^2)$, $aC_7^{\text{eff}} \equiv (2m_b/M_B)C_7^{\text{eff}}$, and the $\mathcal{O}(\alpha_s)$ correction $C^{(1)}$; (2) hard-spectator-scattering term, which includes weak annihilation $T^{(0)}$ (with the main source of strong phase underlined) and $\mathcal{O}(\alpha_s)$ hard-gluon-exchange $T^{(1)}$ terms. The “sum” represents the numerical values of $C_{9,P}^{(t,u)}$ themselves. Following the argument of Ref. [15] for $C_{9,\parallel}^{(t,u)}$, we do not include $1/m_b$ corrections to the second category. See Appendix A for details.

	$q^2 = 2 \text{ GeV}^2$		$q^2 = 5 \text{ GeV}^2$	
	$C_{9,P}^{(t)}$	$C_{9,P}^{(u)}$	$C_{9,P}^{(t)}$	$C_{9,P}^{(u)}$
C_9	4.26	0	4.26	0
$Y(q^2)$	$0.38 + 0.06i$	$-0.50 - 0.85i$	$0.44 + 0.06i$	$-0.17 - 0.85i$
aC_7^{eff}	-0.52	0	-0.52	0
$C^{(1)}$	$-0.24 + 0.01i$	$0.24 + 0.77i$	$-0.27 - 0.01i$	$0.03 + 0.66i$
$T^{(0)} (\pi^-)$	$0.03 - 0.08i$	$1.06 - \underline{2.58i}$	$0.03 - 0.02i$	$1.01 - 0.67i$
(π^0)	$-0.02 + 0.04i$	$0.11 - 0.26i$	$-0.02 + 0.01i$	$0.10 - 0.07i$
$T^{(1)} (\pi^-)$	$0.03 - 0.01i$	$-0.08 - 0.03i$	$0.02 - 0.01i$	$-0.04 - 0.02i$
(π^0)	$0.01 - 0.01i$	$-0.05 + 0.00i$	$0.01 - 0.01i$	$-0.04 - 0.01i$
sum (π^-)	$3.95 - 0.06i$	$0.73 - 2.69i$	$3.97 + 0.03i$	$0.84 - 0.88i$
(π^0)	$3.87 + 0.10i$	$-0.20 - 0.34i$	$3.91 + 0.05i$	$-0.08 - 0.26i$

Eq. (A7), where $\lambda_{B,-}^{-1}$ supplies the source of the imaginary part.

This last effect arises from the matrix element of the operator \mathcal{O}_2^u , which is generated by the tree-level weak annihilation process $\bar{b}u \rightarrow W^* \rightarrow \bar{d}u$ occurring inside the B^+ meson, which is illustrated in Fig. 1. At leading order in $1/m_b$, only the diagram where the photon is emitted by the spectator u -quark in B^+ contributes, and the on-shell intermediate u -quark, preferring smaller q^2 , is responsible for the imaginary part in $\lambda_{B,-}^{-1}$. Because of the tree-level nature, this can be numerically large, as can be read from the large Wilson coefficient $C_2 \sim 1$ in Table III. At larger q^2 , $\lambda_{B,-}^{-1}$ given in Eq. (21) becomes suppressed, and the imaginary part becomes smaller. But the on-shell $q\bar{q}$ ($q = u, d, s$) intermediate states can still offer nonzero imaginary parts at sufficiently large q^2 , as can be read from the imaginary part of $Y(q^2)$ [more precisely

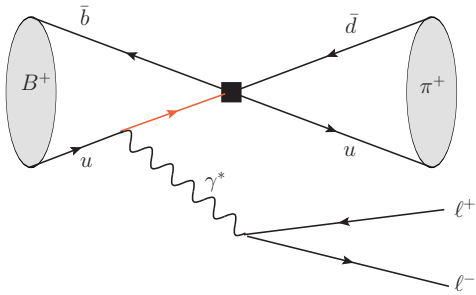


FIG. 1. Tree-level weak annihilation process, $\bar{b}u \rightarrow W^* \rightarrow \bar{d}u$ occurring inside the B^+ meson, generating a large imaginary part through the on-shell u quark (in red).

$Y^{(u)}(q^2)$] and $C^{(1)}$ at $q^2 = 5 \text{ GeV}^2$ in Table IV.

Using the central values and errors for input parameters in Table II, we obtain a prediction for the integrated branching ratio of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ as

$$\begin{aligned}
 & \int_{2 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\mathcal{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)}{dq^2} \\
 &= \left(0.44_{-0.02}^{+0.03} \Big|_{\text{CKM}} \right)_{-0.10}^{+0.13} \Big|_{\text{had.}} \Big|_{-0.01}^{+0.02} \Big|_{\mu} \times 10^{-8} \\
 &= (0.44_{-0.10}^{+0.13}) \times 10^{-8}, \quad (22)
 \end{aligned}$$

where the dominant errors from CKM parameters, hadronic parameters and scale uncertainty are added in quadrature. The integration range of $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$, is for better theoretical control, as explained in previous section. This result and the predictions for the other three decay modes are given in Table V.

The theoretical error is dominated by hadronic uncertainty, which is in turn dominated by uncertainty in form factor normalization $\xi_\pi(0)$. However, this can be largely removed by taking the ratio with the well-measured $B \rightarrow \pi \ell \nu$ rate, given by

$$\begin{aligned}
 \mathcal{B}_{\pi \ell \nu} &\equiv \mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) \\
 &= \frac{\tau_{B^0} G_F^2 |V_{ub}|^2 M_B^3}{192 \pi^3} \int_{q_i^2}^{q_f^2} dq^2 \lambda(q^2, m_{\pi^-}^2)^3 \xi_\pi(q^2)^2. \quad (23)
 \end{aligned}$$

One then obtains the improved decay spectrum,

$$\frac{d\hat{\mathcal{B}}(\bar{B} \rightarrow \pi \ell^+ \ell^-)}{dq^2} \equiv \frac{d\mathcal{B}(\bar{B} \rightarrow \pi \ell^+ \ell^-)/dq^2}{\mathcal{B}_{\pi \ell \nu}} \mathcal{B}_{\pi \ell \nu}^{\text{exp}}. \quad (24)$$

Given the limited range of validity for QCD light-cone sum rule calculations of the form factor, following

TABLE V. Integrated branching ratios of $B \rightarrow \pi \ell \ell$ in unit of 10^{-8} for $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$, obtained in two different ways: “original” definition using Eq. (10); “improved” formula of Eq. (24), by taking the ratio with the $B \rightarrow \pi \ell \nu$ rate. The scale uncertainty (denoted with subscript μ) is estimated by varying the scale $\mu \in [m_b/2, 2m_b]$.

	Original			Improved		
$B^+ \rightarrow \pi^+ \ell^+ \ell^-$	$0.44^{+0.03}_{-0.02}$	$^{+0.13}_{\text{CKM}-0.10}$	$^{+0.02}_{\text{had.}-0.01}$	$0.47^{+0.05}_{-0.03}$	$^{+0.01}_{\text{CKM}-0.01}$	$^{+0.02}_{\text{had.}-0.01}$
$B^- \rightarrow \pi^- \ell^+ \ell^-$	$0.34^{+0.03}_{-0.02}$	$^{+0.11}_{\text{CKM}-0.08}$	$^{+0.02}_{\text{had.}-0.02}$	$0.36^{+0.04}_{-0.03}$	$^{+0.01}_{\text{CKM}-0.01}$	$^{+0.02}_{\text{had.}-0.02}$
$B^0 \rightarrow \pi^0 \ell^+ \ell^-$	$0.18^{+0.01}_{-0.01}$	$^{+0.06}_{\text{CKM}-0.04}$	$^{+0.01}_{\text{had.}-0.01}$	$0.19^{+0.02}_{-0.01}$	$^{+0.00}_{\text{CKM}-0.00}$	$^{+0.01}_{\text{had.}-0.01}$
$\bar{B}^0 \rightarrow \pi^0 \ell^+ \ell^-$	$0.17^{+0.01}_{-0.01}$	$^{+0.05}_{\text{CKM}-0.04}$	$^{+0.01}_{\text{had.}-0.01}$	$0.18^{+0.02}_{-0.01}$	$^{+0.00}_{\text{CKM}-0.00}$	$^{+0.01}_{\text{had.}-0.01}$

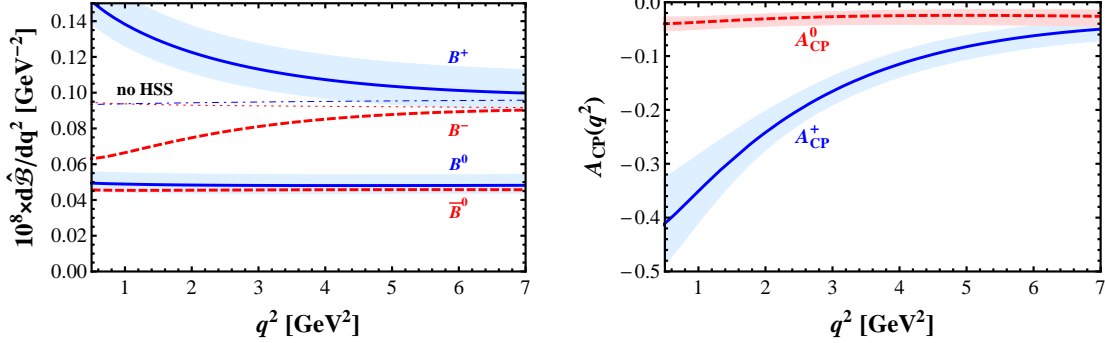


FIG. 2. Differential branching ratios $d\hat{B}/dq^2$ [left] and direct CP asymmetries $A_{\text{CP}}(q^2)$ [right] for $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ and $\bar{B}^0(B^0) \rightarrow \pi^0 \ell^+ \ell^-$, where shaded bands denote theoretical uncertainty. In the left panel, the blue-solid (red-dashed) curves describe B^+ or B^0 (B^- or \bar{B}^0) decay, and “no HSS” indicates results without hard-spectator-scattering terms, where blue dot-dashed (red dotted) curve is for B^+ (B^-). The improved formula of Eq. (24), which takes the ratio with the $B \rightarrow \pi \ell \nu$ rate, is used.

Ref. [30], we take $0 < q^2 < 12 \text{ GeV}^2$ as the integration range for the $B \rightarrow \pi \ell \nu$ rate, and adopt the corresponding HFAG average [36],

$$\begin{aligned} \mathcal{B}_{\pi \ell \nu}^{\text{exp}} &\equiv \mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell)_{q^2 < 12 \text{ GeV}^2}^{\text{exp}} \\ &= (0.81 \pm 0.02|_{\text{stat.}} \pm 0.03|_{\text{syst.}}) \times 10^{-4}. \end{aligned} \quad (25)$$

Using Eq. (24), we then obtain the improved integrated branching ratio of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ as

$$\begin{aligned} &\int_{2 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\hat{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)}{dq^2} \\ &= \left(0.47^{+0.05}_{-0.03} \Big|_{\text{CKM}-0.01}^{+0.01} \Big|_{\text{had.}-0.01}^{+0.02} \Big|_{\mu-0.02}^{+0.02} \Big|_{\pi \ell \nu} \right) \times 10^{-8} \\ &= (0.47^{+0.06}_{-0.04}) \times 10^{-8}. \end{aligned} \quad (26)$$

Indeed, the hadronic uncertainty is reduced considerably by cancelation of the form factor normalization $\xi_\pi(0)$, at the cost of introducing an extra but moderate error from $\mathcal{B}_{\pi \ell \nu}^{\text{exp}}$ (denoted by subscript $\pi \ell \nu$). Also, the error from CKM parameters gets slightly enhanced due to $|V_{ub}|^2$, which is brought in with the $B \rightarrow \pi \ell \nu$ rate. As a whole, the total error is reduced down to 10% level from the 20–30% error in the original case.

The central value of Eq. (26) is slightly raised from Eq. (22). Given our integration range of 2–6 GeV^2 is smaller than the full $0 \lesssim q^2 \lesssim 26.4 \text{ GeV}^2$ range by

more than a factor six, it is comforting to see that our value is smaller than the LHCb value [Eq. (1)] by a factor five. Our prediction should be checked by LHCb with full 2011–2012 dataset, as well as Run II and future data.

The improved predictions for all four decay modes are also given in Table V. The $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ rate is roughly a factor of two smaller than $B^+ \rightarrow \pi^+ \ell^+ \ell^-$, mainly due to the isospin factor $S_{\pi^0}/S_{\pi^+} = 1/2$. Results for $q^2 \in (1, 6) \text{ GeV}^2$ and $(1, 8) \text{ GeV}^2$ will be given later for comparison with literature.

The q^2 spectra of the four $B \rightarrow \pi \ell^+ \ell^-$ modes are shown in the left panel of Fig. 2. The decay spectrum for $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ is almost flat in the range shown, with small difference between the CP-conjugate modes. In contrast, the decay spectra for $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ show visible q^2 dependence. At high q^2 , the B^+ and B^- differential rates are similar, and larger than B^0 case by roughly a factor two due to isospin. However, at low q^2 , the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ rate tends larger while $B^- \rightarrow \pi^- \ell^+ \ell^-$ tends smaller, which signals direct CP asymmetry.

For comparison, we also give in the left panel of Fig. 2 the B^+ and B^- differential rates with the hard-spectator-scattering terms removed. Similar to the prediction in Ref. [7], the two decay spectra are almost degenerate and rather flat. This shows that hard-spectator-scattering, especially weak annihilation, is the main source for differ-

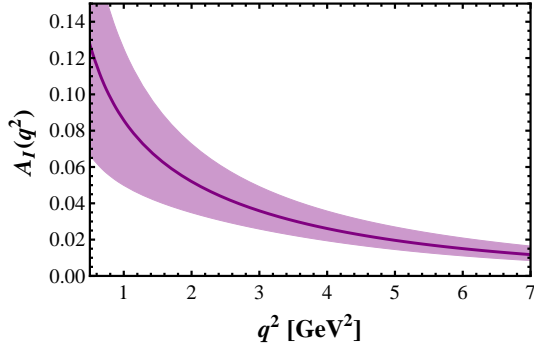


FIG. 3. Isospin asymmetry $A_I(q^2)$ for $B \rightarrow \pi \ell^+ \ell^-$ decays, with shaded area indicating theoretical errors.

ence between B^+ and B^- differential rates, hence direct CP asymmetry, in low q^2 region.

The q^2 -dependence of the direct CP asymmetries for $B^{+,0} \rightarrow \pi^{+,0} \ell^+ \ell^-$ are given in the right panel of Fig. 2. Indeed, the strength of $A_{\text{CP}}^+(q^2)$ for $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ gets larger at lower q^2 , reaching -40% , while $A_{\text{CP}}^0(q^2)$ for $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ stays almost flat between -2% to -4% . Although we imposed $q^2 > 2 \text{ GeV}^2$ for better theoretical control, $A_{\text{CP}}^+(q^2)$ can still reach -25% . For sake of showing the trend, we have plotted both $d\hat{B}/dq^2$ and $A_{\text{CP}}(q^2)$ in Fig. 2 beyond our conservative range of 2 to 6 GeV^2 .

We note that the CP asymmetry is proportional to $\text{Im}(\lambda_t^* \lambda_u) \text{Im}(\mathcal{C}_{9,P}^{(t)*} \mathcal{C}_{9,P}^{(u)})$, where $\text{Im}(\lambda_t^* \lambda_u) \propto \sin \phi_2$. Besides nonzero “weak phase” ϕ_2 , we also need nonvanishing phase between $\mathcal{C}_{9,P}^{(u)}$ and $\mathcal{C}_{9,P}^{(t)}$, i.e. “strong phase”, for generating A_{CP} . The large imaginary part of $\mathcal{C}_{9,P}^{(u)}$ comes from the weak annihilation term (see Fig. 1), especially for low q^2 , with subdominant effect from real $q\bar{q}$ intermediate states, which can be read from Table IV. On the other hand, $\mathcal{C}_{9,P}^{(t)}$ is dominated by the real valued \mathcal{C}_9 even for low q^2 . This results in net strong phase difference and therefore large CP asymmetry at low q^2 .

The CP asymmetries averaged over our q^2 range are

$$\begin{aligned} \langle A_{\text{CP}}^+ \rangle &= -0.13_{-0.01}^{+0.01} \Big|_{\text{CKM}} \Big|_{-0.02}^{+0.02} \Big|_{\text{had.}} \Big|_{-0.02}^{+0.01} \Big|_{\mu} \\ &= -0.13_{-0.03}^{+0.02}, \end{aligned} \quad (27)$$

$$\begin{aligned} \langle A_{\text{CP}}^0 \rangle &= -0.03_{-0.00}^{+0.00} \Big|_{\text{CKM}} \Big|_{-0.00}^{+0.00} \Big|_{\text{had.}} \Big|_{-0.02}^{+0.01} \Big|_{\mu} \\ &= -0.03_{-0.02}^{+0.01}, \end{aligned} \quad (28)$$

which can be obtained from Table V directly. Because of averaging over a q^2 range, the rather large CP asymmetry at low q^2 for $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ gets diluted. But the LHCb experiment could target the low q^2 region, even below 2 GeV^2 , to zoom in on the effect.

The CP asymmetry for $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ is small, as “weak annihilation” is mediated by color-suppressed W -exchange process $\bar{b}d \rightarrow \bar{u}u$ or loop-suppressed QCD penguins, while the charge of the spectator quark leads to a factor of 2 reduction. Measuring this small asymmetry would be challenging, even for the Belle II experiment.

The direct CP asymmetries for $B \rightarrow K \ell^+ \ell^-$ can arise via similar mechanism, but is highly suppressed by the hierarchy of CKM factors, $|V_{us}^* V_{ub}| \ll |V_{ts}^* V_{tb}|$. This is consistent with non-observation of CP asymmetry in this mode reported recently by LHCb [23].

The q^2 -dependence of the isospin asymmetry A_I is given in Fig. 3. While it gets larger at lower q^2 , the asymmetry is far below 10%. A_I can also be generated by hard-spectator scatterings, as in the case for CP asymmetries. However, the effect is reduced since we take CP average in Eq. (14), which smooths the oppositely varying B^+ and B^- decay rates at lower q^2 . We obtain the q^2 -averaged isospin asymmetry in our range as

$$\begin{aligned} \langle A_I \rangle &= 0.03_{-0.00}^{+0.01} \Big|_{\text{CKM}} \Big|_{-0.01}^{+0.01} \Big|_{\text{had.}} \Big|_{-0.00}^{+0.00} \Big|_{\mu} \\ &= 0.03 \pm 0.01. \end{aligned} \quad (29)$$

Measurement of the isospin asymmetry depends on measuring $B^0 \rightarrow \pi^0 \ell^+ \ell^-$, but appears rather difficult.

For sake of comparison with literature, we give in Table VI predictions for various observables for $q^2 \in (1, 6) \text{ GeV}^2$ and $(1, 8) \text{ GeV}^2$, side by side with our adopted range of $q^2 \in (2, 6) \text{ GeV}^2$. We note that our error estimate does not take into account $1/m_b$ corrections nor systematic errors due to use of model functions for the B -meson light-cone distribution amplitudes. Some discussion on model dependence is deferred to Appendix B.

IV. CONSTRAINING CKM PARAMETERS

So far we have assumed the Wolfenstein parameters determined by global fit, listed in Table II, as CKM inputs. However, once the $B \rightarrow \pi \ell^+ \ell^-$ observables are measured with some accuracy, one may use the measured values to determine the CKM parameters assuming SM. In particular, the $B \rightarrow \pi \ell^+ \ell^-$ observables are sensitive to $\phi_2 \equiv \alpha$,

TABLE VI. SM predictions in three different q^2 -ranges for improved branching ratios (in units of 10^{-8}), q^2 -averaged CP and isospin asymmetries. The R_+ -related quantities F_+^2 , c_+ and d_+ (Eq. (32)) are also given. Only total errors are shown.

	(2, 6) GeV^2	(1, 6) GeV^2	(1, 8) GeV^2
$\hat{B}(B^+ \rightarrow \pi^+ \ell \ell)$	$0.47_{-0.04}^{+0.06}$	$0.61_{-0.06}^{+0.07}$	$0.82_{-0.07}^{+0.10}$
$\hat{B}(B^- \rightarrow \pi^- \ell \ell)$	$0.36_{-0.04}^{+0.05}$	$0.44_{-0.05}^{+0.06}$	$0.63_{-0.07}^{+0.09}$
$\hat{B}(B^0 \rightarrow \pi^0 \ell \ell)$	0.19 ± 0.02	$0.24_{-0.02}^{+0.03}$	$0.34_{-0.03}^{+0.04}$
$\hat{B}(\bar{B}^0 \rightarrow \pi^0 \ell \ell)$	0.18 ± 0.02	$0.23_{-0.02}^{+0.03}$	$0.32_{-0.04}^{+0.05}$
$\langle A_{\text{CP}}^+ \rangle$	$-0.13_{-0.03}^{+0.02}$	-0.16 ± 0.02	-0.13 ± 0.02
$\langle A_{\text{CP}}^0 \rangle$	$-0.03_{-0.02}^{+0.01}$	$-0.03_{-0.02}^{+0.01}$	$-0.03_{-0.02}^{+0.01}$
$\langle A_I \rangle$	0.03 ± 0.01	0.04 ± 0.01	0.03 ± 0.01
F_+^2	$0.58_{-0.08}^{+0.09}$	$0.59_{-0.08}^{+0.09}$	$0.60_{-0.08}^{+0.09}$
c_+	$0.25_{-0.06}^{+0.07}$	$0.21_{-0.07}^{+0.08}$	$0.20_{-0.05}^{+0.06}$
d_+	$0.13_{-0.03}^{+0.04}$	0.20 ± 0.06	$0.15_{-0.03}^{+0.04}$

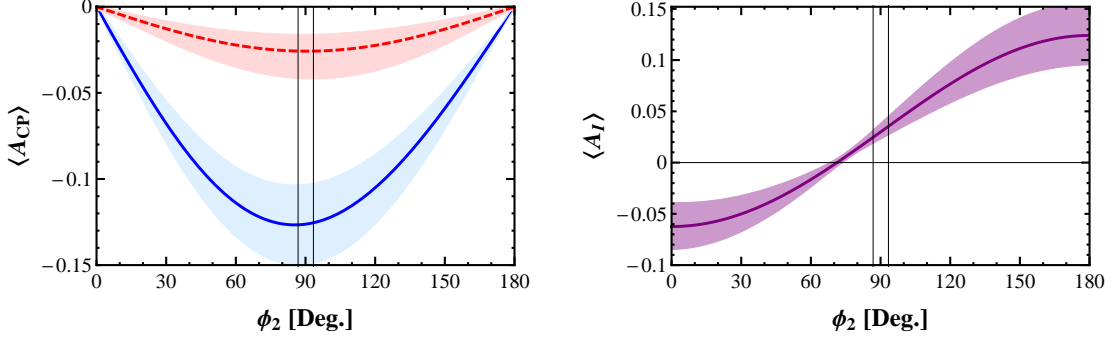


FIG. 4. The ϕ_2 dependence of q^2 -averaged asymmetries for $B \rightarrow \pi \ell^+ \ell^-$ with $R_{ut} = 0.39$: [left] direct CP asymmetries for $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ (blue-solid) and $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ (red-dashed); [right] isospin asymmetry for $B \rightarrow \pi \ell^+ \ell^-$ decays. Shaded regions indicate the combined hadronic and scale uncertainties, and black solid lines indicate the global fit [17] value $\phi_2 = (89_{-2}^{+4})^\circ$.

an angle in the $b \rightarrow d$ unitarity triangle.

We show in Fig. 4 the ϕ_2 dependence of the q^2 -averaged asymmetries with fixed $R_{ut} = |\lambda_u/\lambda_t| = 0.39$ (see Eq. (12) for definition). For $\phi_2 = (89_{-2}^{+4})^\circ$ from global analysis [17], the direct CP asymmetries are close to maximum for both neutral and charged B decays. On the other hand, the isospin asymmetry for this ϕ_2 value is small, located near the vanishing point.

LHCb has also reported [1] the ratio of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ \mu^+ \mu^-$ branching ratios,

$$\frac{\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)} = 0.053 \pm 0.014 \text{ (stat.)} \pm 0.001 \text{ (syst.)}. \quad (30)$$

LHCb utilized this to determine the ratio $|V_{td}|/|V_{ts}|$, with form factors and Wilson coefficients as theoretical input. As the formulas in Sec. II can be straightforwardly applied to $B^+ \rightarrow K^+ \ell^+ \ell^-$ decay, one can calculate the corresponding quantity in low q^2 region based on QCDF.

For our estimate, we define

$$R_+ \equiv \frac{\overline{\mathcal{B}}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)}{\overline{\mathcal{B}}(B^+ \rightarrow K^+ \ell^+ \ell^-)}, \quad (31)$$

where $\overline{\mathcal{B}}$ are CP-averaged branching ratios integrated over $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$. Besides overall $|V_{td}|/|V_{ts}|$ dependence, R_+ also depends on R_{ut} and ϕ_2 through $\overline{\mathcal{B}}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$. More explicitly, CKM dependence can be extracted from

$$R_+ = \left| \frac{V_{td}}{V_{ts}} \right|^2 F_+^2 [1 - c_+ R_{ut} \cos \phi_2 + d_+ R_{ut}^2], \quad (32)$$

which involves the CKM-independent quantities,

$$F_+^2 = \left(\frac{\xi_\pi(0)}{\xi_K(0)} \right)^2 \frac{\langle |C_{9,\pi^-}^{(t)}|^2 + C_{10}^2 \rangle_{\pi^-}}{\langle |C_{9,K^-}^{(t)}|^2 + C_{10}^2 \rangle_{K^-}}, \quad (33)$$

and

$$c_+ = 2 \frac{\langle \text{Re}[C_{9,\pi^-}^{(t)*} C_{9,\pi^-}^{(u)}] \rangle_{\pi^-}}{\langle |C_{9,\pi^-}^{(t)}|^2 + C_{10}^2 \rangle_{\pi^-}}, \quad d_+ = \frac{\langle |C_{9,\pi^-}^{(u)}|^2 \rangle_{\pi^-}}{\langle |C_{9,\pi^-}^{(t)}|^2 + C_{10}^2 \rangle_{\pi^-}}. \quad (34)$$

We have introduced, for a q^2 -dependent quantity $X(q^2)$, an abbreviation

$$\langle X \rangle_P \equiv \int_{2 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \lambda(q^2, m_P^2)^3 \left(\frac{\xi_P(q^2)}{\xi_P(0)} \right)^2 X(q^2). \quad (35)$$

F_+^2 corresponds to f^2 in Ref. [1] with $R_{ut} = 0$, while c_+ and d_+ deform the correspondence when $R_{ut} \neq 0$.

Our numerical result for $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ (results for other q^2 ranges are given in Table VI) is

$$F_+^2 = 0.58_{-0.08}^{+0.09}, \quad c_+ = 0.25_{-0.06}^{+0.07}, \quad d_+ = 0.13_{-0.03}^{+0.04}. \quad (36)$$

For $R_{ut} = 0.39$ and $\phi_2 = 89^\circ$ from global analysis [17], the terms including c_+ or d_+ in Eq. (32) contribute by only a few percent. In this case, the main theoretical uncertainty in Eq. (32) comes from the overall F_+^2 , where the uncertainty is dominated by the one from the form factor ratio $\xi_K(0)/\xi_\pi(0)$. The QCD light-cone sum rule result [33], which takes the SU(3) breaking corrections into account, provides this form factor ratio with better precision than the individual ones (see Table II). Therefore, the error of F_+^2 , which is around 15%, is much smaller than the 30 or 20% error in the original $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ branching ratio, shown in Eq. (22).

Of course, our result can not be directly compared with the LHCb result, where the full q^2 range seems to be used. However, it should become possible once statistics is increased in the near future. Although the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ is rather well measured, this method can provide a complementary check. Once several $B \rightarrow \pi \ell^+ \ell^-$ observables are measured with certain precision, their combination would be useful to constrain the CKM parameters, adding further information for existing studies on the unitarity triangle. In the remaining part of this

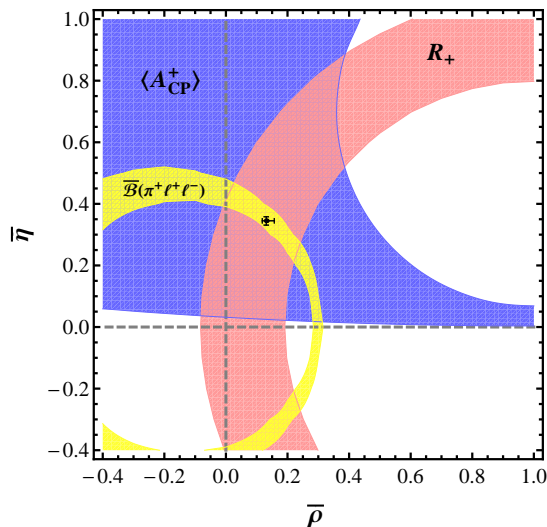


FIG. 5. Possible constraints on $(\bar{\rho}, \bar{\eta})$ plane from CP-averaged $\bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ (improved via Eq. (24)), R_+ and $\langle A_{\text{CP}}^+ \rangle$, assuming the guesstimates given in Eqs. (37) and (38). The allowed regions are drawn by also taking into account theoretical uncertainty (except CKM) for these observables, added linearly with experimental errors. The black dot with error bars indicates preference of global analysis [17].

section, we illustrate the possible impact of future measurements of these observables.

Despite the observation of the $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ mode by LHCb with 1.0 fb^{-1} data [1], it might take a while to establish its branching ratio in our q^2 range, $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$, due to limited statistics. Nevertheless, LHCb will update with full 2011–2012 data, and with Run 2 of the LHC at 13–14 TeV to start in 2015, precise measurements of the $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ branching ratio in our q^2 range may become possible. With this anticipation, we make the following guesstimate for the CP-averaged branching ratio $\bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ and R_+ in our q^2 range, for LHCb with Run 2 data:

$$\begin{aligned} \bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)^{\text{exp}} &= (0.42 \pm 0.04) \times 10^{-8}, \\ R_+^{\text{exp}} &= 0.027 \pm 0.003, \end{aligned} \quad (37)$$

where we take 10% experimental error with SM-like central values. At this precision for the branching ratio, the SM-like value for the q^2 -averaged $\langle A_{\text{CP}}^+ \rangle \sim -10\%$ would still be difficult to establish. We simply assume

$$\langle A_{\text{CP}}^+ \rangle^{\text{exp}} = -0.13 \pm 0.10, \quad (38)$$

but refinement, especially zooming in on larger asymmetry at lower q^2 , can be done by LHCb.

Possible constraints on the $(\bar{\rho}, \bar{\eta})$ plane with these projections for future measurements are illustrated in Fig. 5. $\bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ [improved via Eq. (24)] provides a narrower allowed ring than R_+ , because of smaller theoretical uncertainty as well as stronger dependence on $\bar{\rho}$ and $\bar{\eta}$, brought about by $V_{ub} = A\lambda^3(\rho - i\eta)$ from

$B \rightarrow \pi \ell \nu$ rate. One notes that the combined constraint of $\bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ and R_+ allows two strip-shaped regions. If $\langle A_{\text{CP}}^+ \rangle$ is found with negative sign, as in Eq. (38), it would favor the upper strip. Note that $\langle A_{\text{CP}}^+ \rangle$ changes sign around $\bar{\eta} = 0$: $\langle A_{\text{CP}}^+ \rangle < 0$ for $\bar{\eta} > 0$ and vice versa. Therefore, even though establishing the value of $\langle A_{\text{CP}}^+ \rangle$ in the near future may be challenging, measuring the sign of $\langle A_{\text{CP}}^+ \rangle$ would still provide useful information to discriminate between the two separate regions.

For the neutral B decay case, so far only upper limits are available (see Table I). The Belle II experiment may improve the situation in the future. But judging from the poor performance of $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ by Belle, as indicated in Table I, there is some worry whether this rate can be measured well for Belle II. The situation for $\langle A_{\text{CP}}^0 \rangle$ and $\langle A_I \rangle$ would be much worse due to the small SM predictions. However, even if measurements for $B^0 \rightarrow \pi^0 \ell^+ \ell^-$ is not forthcoming, the charged B decay observables by themselves can still provide useful information, as we have explained. We do not show possible constraints by neutral B decay related observables.⁴

V. DISCUSSION AND CONCLUSION

Observation of the rare decay $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ indicates that the $b \rightarrow d \ell^+ \ell^-$ precision era has dawned. As a preparation for the advent of more precise measurements, our systematic study of the semileptonic decay $B \rightarrow \pi \ell^+ \ell^-$ based on the QCDF framework suggests rich information, and sizable direct CP asymmetry is predicted for charged B decay for $q^2 \lesssim 3\text{--}4 \text{ GeV}^2$. Such large CP asymmetry could be background of the CP asymmetry measurement in $B^+ \rightarrow K^+ \mu^+ \mu^-$ decays, which seems to be glossed over in the LHCb study [23].

We advocate the q^2 region between 2 to 6 GeV^2 , where QCDF is applicable. Hadronic uncertainty due to form factors are further reduced by taking ratio with $B^0 \rightarrow \pi^- \ell^+ \nu$ rate. We have shown, due to large absorptive part arising from hard-spectator-scattering, i.e. the annihilation diagram of Fig. 1, together with the CP phase via V_{ub} and V_{td} , leads to a sizable CP asymmetry A_{CP}^+ in decay rate of $B^+ \rightarrow \pi^+ \ell^+ \ell^-$, which grows for lower q^2 . Similar effect is suppressed for $B^0 \rightarrow \pi^0 \ell^+ \ell^-$, which has a rather small direct CP asymmetry. We also study the isospin asymmetry A_I between the charged and neutral B decays. While it also grows with lower q^2 , the effect remains below 10% and smaller than A_{CP}^+ . Given that it would require the measurement of $B^0 \rightarrow \pi^0 \ell^+ \ell^-$, which can only be done by Belle II, its measurement is more distant in the future.

⁴ We have checked that constraints from $\bar{B}(B^0 \rightarrow \pi^0 \ell^+ \ell^-)$ assuming 10% experimental error provide similar allowed region as $\bar{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ in Fig. 5. Such measurements, however, do not seem reachable in the foreseeable future.

To emphasize the importance and utility of further, improved measurements of $B^+ \rightarrow \pi^+\ell^+\ell^-$, we illustrate its potential for crosschecking the three generation SM, by showing the intersect of the allowed region of $B^+ \rightarrow \pi^+\ell^+\ell^-$ rate normalized by $B^0 \rightarrow \pi^-\ell^+\nu$, as well as normalized by $B^+ \rightarrow K^+\ell^+\ell^-$, which bring in different CKM dependence. Assuming some future precision with the SM-like central values, we show that it has two intersecting regions. Interestingly, by measuring the *sign* of A_{CP}^+ (rather than a more precise value measurement), one can eliminate one of the regions, hence provide a measure on the $(\bar{\rho}, \bar{\eta})$ plane. Though not spectacular, it provides independent information from, e.g. B_d over B_s mixing.

The direct CP asymmetry gets enhanced at low q^2 , a firm consequence of the annihilation diagram that participates to $B^+ \rightarrow \pi^+\ell^+\ell^-$ at tree level. Since the trend continues as q^2 gets lower, we have explored below 2 GeV², beyond the region of applicability for QCDF, and entering the hadronic resonance region. With increased data, LHCb should target this low q^2 region, below 3–4 GeV², to check this enhancement. This not only facilitates the CKM measurement program, but also checks the presence of the annihilation diagram.

The annihilation diagram provides a tree level mechanism for bringing in a large strong phase through the on-shell u quark from the B^+ wavefunction (or light-cone distribution amplitude), by emitting a photon that becomes an $\ell^+\ell^-$ pair. The strength of A_{CP}^+ reaches beyond –25% for q^2 around 2 GeV², and the trend continues as q^2 is lowered. Recently, the LHCb experiment has measured CP asymmetries in 3-body $B^\pm \rightarrow K^\pm h^+ h^-$ [39] and $B^\pm \rightarrow \pi^\pm h^+ h^-$ [40] decays, where $h^+ h^-$ stands for $\pi^+\pi^-$ or K^+K^- . All modes give large “regional” CP asymmetries, with absolute values reaching beyond 50%, except the $K^\pm K^+ K^-$ final state. The CP asymmetry Dalitz plots for $B^\pm \rightarrow K^\pm h^+ h^-$ and $\pi^\pm \pi^+ \pi^-$ exhibit complex structures, and contain a lot of hadronic effects. Of particular interest to us is $B^\pm \rightarrow \pi^\pm K^+ K^-$, where LHCb finds CP asymmetry as large as $\sim -60\%$ for $m_{K^+ K^-}^2 < 1.5$ GeV², which is reminiscent of our finding for $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ decay. Indeed, replacing $K^+ K^-$ by $\ell^+ \ell^-$ gives the $B^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ process. Could the observed effect occur through a similar mechanism as we discussed? It certainly cannot occur through Fig. 1 with a virtual photon, as α is way too weak compared to hadronic couplings. However, we speculate that, if the virtual photon is replaced by some hadron that couples to both $u\bar{u}$ and $s\bar{s}$, a similar effect should be achievable, and the absorptive part can come from u quark being on-shell in Fig. 1, rather than from the resonance width. We will return to a model study of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ for low $m_{K^+ K^-}^2$ in a future work.

In conclusion, we have studied $B^{+,0} \rightarrow \pi^{+,0} \ell^+ \ell^-$ decays in the framework of QCDF, giving the SM prediction for the branching ratios in the $q^2 = m_{\ell^+ \ell^-}^2$ range of 2 to 6 GeV², together with a slightly larger range for comparison. We study also the associated CP and isospin

asymmetries. A relatively precise prediction is made for $\mathcal{B}(B^+ \rightarrow \pi^+ \ell^+ \ell^-)$ that can be tested by experiment. Furthermore, we find that, due to annihilation diagram with hard-spectator scattering, the CP asymmetry grows with lower q^2 and can become rather sizable, which is another feature that can be tested by experiment. Future precision measurements can provide a check on CKM parameters. Our mechanism for large CP asymmetries at low q^2 seems to echo the large CP asymmetry for low $m_{K^+ K^-}^2$ observed by LHCb for $B^\pm \rightarrow \pi^\pm K^+ K^-$.

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Appendix A: $\bar{B} \rightarrow P\gamma^*$ amplitude

As already given in Eq. (7), the $\bar{B} \rightarrow P\gamma^*$ amplitude at $\mathcal{O}(\alpha_s)$ in the heavy quark limit is given by

$$\mathcal{T}_P^{(i)} = \xi_P C_P^{(i)} + \zeta_P \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \int_0^1 du \phi_P(u) T_{P,\pm}^{(i)}(u, \omega) \quad (\text{A1})$$

where

$$\zeta_P = \frac{\pi^2}{N_c} \frac{f_B f_P}{M_B}, \quad (\text{A2})$$

$$C_P^{(i)} = C_P^{(0,i)} + \frac{\alpha_s C_F}{4\pi} C_P^{(1,i)}, \quad (\text{A3})$$

$$T_{P,\pm}^{(i)}(u, \omega) = T_{P,\pm}^{(0,i)}(u, \omega) + \frac{\alpha_s C_F}{4\pi} T_{P,\pm}^{(1,i)}(u, \omega), \quad (\text{A4})$$

and $i = t, u$. One can show that the $\bar{B} \rightarrow P\gamma^*$ amplitude is related to the $\bar{B} \rightarrow V_\parallel \gamma^*$ amplitude, where V_\parallel is longitudinally polarized, through

$$C_P^{(i)} = -C_\parallel^{(i)}, \quad T_{P,\pm}^{(i)}(u, \omega) = -T_{\parallel,\pm}^{(i)}(u, \omega). \quad (\text{A5})$$

Therefore, one can utilize the existing result for $\bar{B} \rightarrow \rho\gamma^*$ [15] to obtain the $\bar{B} \rightarrow \pi\gamma^*$ amplitudes. The explicit expressions for the $\bar{B} \rightarrow P\gamma^*$ amplitudes are

$$\begin{aligned} \mathcal{T}_P^{(t)} = & \xi_P \left(C_P^{(0,t)} + \frac{\alpha_s C_F}{4\pi} [C_P^{(f,t)} + C_P^{(nf,t)}] \right) \\ & + \zeta_P \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(0,t)} \\ & + \frac{\alpha_s C_F}{4\pi} \zeta_P \left(\lambda_{B,+}^{-1} \int du \phi_P(u) [T_{P,+}^{(f,t)}(u) + T_{P,+}^{(nf,t)}(u)] \right. \\ & \left. + \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(nf,t)}(u) \right), \quad (\text{A6}) \end{aligned}$$

$$\begin{aligned}
\mathcal{T}_P^{(u)} = & \xi_P \left(C_P^{(0,u)} + \frac{\alpha_s C_F}{4\pi} C_P^{(nf,u)} \right) \\
& + \zeta_P \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(0,u)} \\
& + \frac{\alpha_s C_F}{4\pi} \zeta_P \left(\lambda_{B,+}^{-1} \int du \phi_P(u) T_{P,+}^{(nf,u)}(u) \right. \\
& \left. + \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(nf,u)}(u) \right), \quad (A7)
\end{aligned}$$

where we have introduced

$$\begin{aligned}
\hat{T}_{P,-}^{(0,i)} & \equiv \frac{M_B \omega - q^2 - i\epsilon}{M_B \omega} T_{P,-}^{(0,i)}(u, \omega), \\
\hat{T}_{P,-}^{(nf,i)}(u) & \equiv \frac{M_B \omega - q^2 - i\epsilon}{M_B \omega} T_{P,-}^{(nf,i)}(u, \omega), \quad (A8)
\end{aligned}$$

to remove ω dependence.

The $\mathcal{O}(\alpha_s^0)$ contributions to the form factor term are

$$\begin{aligned}
C_P^{(0,t)} & = C_7^{\text{eff}} + \frac{M_B}{2m_b} Y(q^2), \\
C_P^{(0,u)} & = \frac{M_B}{2m_b} Y^{(u)}(q^2), \quad (A9)
\end{aligned}$$

while the $\mathcal{O}(\alpha_s)$ factorizable and nonfactorizable corrections are ($C_F = 4/3$)

$$C_P^{(f,t)} = \left(\ln \frac{m_b^2}{\mu^2} + 2L + \Delta M \right) C_7^{\text{eff}}, \quad (A10)$$

$$\begin{aligned}
C_F C_P^{(nf,t)} = & -\bar{C}_2 F_2^{(7)} - C_8^{\text{eff}} F_8^{(7)} \\
& - \frac{M_B}{2m_b} \left[\bar{C}_2 F_2^{(9)} + 2\bar{C}_1 \left(F_1^{(9)} + \frac{1}{6} F_2^{(9)} \right) \right. \\
& \left. + C_8^{\text{eff}} F_8^{(9)} \right], \quad (A11)
\end{aligned}$$

$$\begin{aligned}
C_F C_P^{(nf,u)} = & -\bar{C}_2 \left(F_2^{(7)} + F_{2,u}^{(7)} \right) \\
& - \frac{M_B}{2m_b} \left[\bar{C}_2 \left(F_2^{(9)} + F_{2,u}^{(9)} \right) \right. \\
& \left. + 2\bar{C}_1 \left(\left(F_1^{(9)} + F_{1,u}^{(9)} \right) + \frac{1}{6} \left(F_2^{(9)} + F_{2,u}^{(9)} \right) \right) \right]. \quad (A12)
\end{aligned}$$

The $\mathcal{O}(\alpha_s^0)$ contributions to hard-spectator-scattering term from weak annihilation diagrams of Fig. 1 are

$$\hat{T}_{P,-}^{(0,t)} = e_q \frac{4M_B}{m_b} C_q^{34}, \quad \hat{T}_{P,-}^{(0,u)} = -e_q \frac{4M_B}{m_b} C_q^{12}, \quad (A13)$$

where $q = u, d$ is the flavor of the spectator quark in the B meson with e_q ($q = u, d$) its charge, and

$$C_q^{34} \equiv C_3 + \frac{4}{3} (C_4 + 12C_5 + 16C_6), \quad (A14)$$

$$C_q^{12} \equiv 3\delta_{qu} C_2 - \delta_{qd} \left(\frac{4}{3} C_1 + C_2 \right). \quad (A15)$$

The $\mathcal{O}(\alpha_s)$ factorizable and nonfactorizable corrections to the hard-spectator-scattering term are given by

$$T_{P,+}^{(f,t)}(u) = -C_7^{\text{eff}} \frac{4M_B}{\bar{u}E}, \quad (A16)$$

$$\begin{aligned}
T_{P,+}^{(nf,t)}(u) = & -\frac{M_B}{m_b} \left[e_u t_{\parallel}(u, m_c) (\bar{C}_2 + \bar{C}_4 - \bar{C}_6) \right. \\
& \left. + e_d t_{\parallel}(u, m_b) (\bar{C}_3 + \bar{C}_4 - \bar{C}_6) + e_d t_{\parallel}(u, 0) \bar{C}_3 \right], \quad (A17)
\end{aligned}$$

$$\begin{aligned}
T_{P,+}^{(nf,u)}(u) = & -e_u \frac{M_B}{m_b} \left(C_2 - \frac{1}{6} C_1 \right) \\
& \times [t_{\parallel}(u, m_c) - t_{\parallel}(u, 0)], \quad (A18)
\end{aligned}$$

$$\begin{aligned}
\hat{T}_{P,-}^{(nf,t)}(u) = & -e_q \left[\frac{8C_8^{\text{eff}}}{\bar{u} + uq^2/M_B^2} \right. \\
& + \frac{6M_B}{m_b} \left\{ h(\bar{u}M_B^2 + uq^2, m_c) (\bar{C}_2 + \bar{C}_4 + \bar{C}_6) \right. \\
& + h(\bar{u}M_B^2 + uq^2, m_b) (\bar{C}_3 + \bar{C}_4 + \bar{C}_6) \\
& + h(\bar{u}M_B^2 + uq^2, 0) (\bar{C}_3 + 3\bar{C}_4 + 3\bar{C}_6) \\
& \left. \left. - \frac{8}{27} (\bar{C}_3 - \bar{C}_5 - 15\bar{C}_6) \right\} \right], \quad (A19)
\end{aligned}$$

$$\begin{aligned}
\hat{T}_{P,-}^{(nf,u)}(u) = & -e_q \frac{6M_B}{m_b} \left(C_2 - \frac{1}{6} C_1 \right) \\
& \times [h(\bar{u}M_B^2 + uq^2, m_c) - h(\bar{u}M_B^2 + uq^2, 0)], \quad (A20)
\end{aligned}$$

where \bar{C}_i (for $i = 1, \dots, 6$) are defined by

$$\begin{aligned}
\bar{C}_1 & = \frac{1}{2} C_1, \quad \bar{C}_2 = C_2 - \frac{1}{6} C_1, \\
\bar{C}_3 & = C_3 - \frac{1}{6} C_4 + 16C_5 - \frac{8}{3} C_6, \\
\bar{C}_4 & = \frac{1}{2} C_4 + 8C_6, \\
\bar{C}_5 & = C_3 - \frac{1}{6} C_4 + 4C_5 - \frac{2}{3} C_6, \\
\bar{C}_6 & = \frac{1}{2} C_4 + 2C_6. \quad (A21)
\end{aligned}$$

Details of the other definitions can be found in Refs. [12, 15], and references therein.

Appendix B: Hadronic model dependence

We comment that the larger error at lower q^2 for asymmetries A_{CP} and A_1 is due to the dominance of weak annihilation. As the latter is calculated at leading order in $1/m_b$ [12], there might be $1/m_b$ corrections even at $\mathcal{O}(\alpha_s^0)$, whose uncertainty is not included in our error estimate. Besides, the use of model functions for B -meson light-cone distribution amplitudes might introduce extra uncertainty, which we will turn to shortly. For the pion light-cone distribution amplitude ϕ_π , its uncertainty has little effect on the hadronic errors, although the Gegenbauer coefficients $a_{2,4}^\pi$ listed in Table II are rather uncertain. This is because the leading order weak annihilation

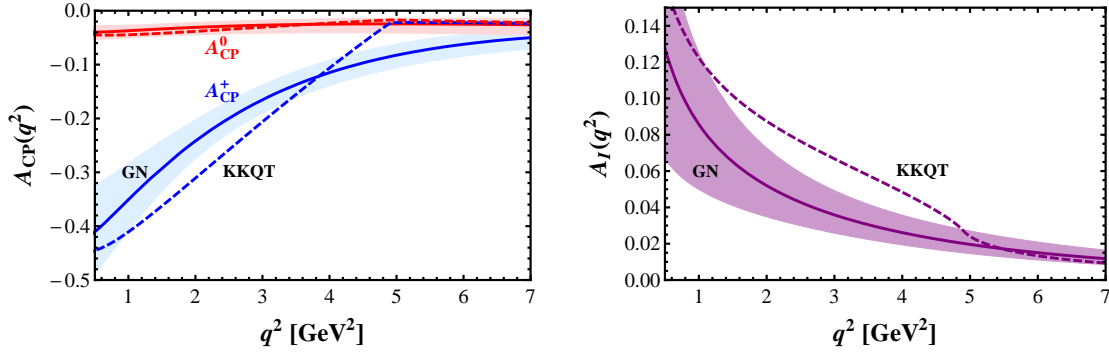


FIG. 6. $A_{CP}(q^2)$ [left] and $A_I(q^2)$ [right] obtained by using “KKQT” B -meson light-cone distribution amplitudes [41] (dashed), compared with the “GN” model (solid) result given in the right panel of Fig. 2 and Fig. 3, where shaded bands denote theoretical uncertainty of the GN model result.

$\hat{T}_{P,-}^{(0,i)}$ that enters Eqs. (A6) and (A7) does not depend on u . Thus, $\phi_\pi(u)$ can be integrated out, giving simply $\int du \phi_\pi = 1$ due to normalization. Therefore, there is no dependence on the Gegenbauer coefficients in the leading order weak annihilation terms.

To see the impact of shape functions for B -meson light-cone amplitudes, we reevaluate previous results using

$$\begin{aligned}\Phi_{B,+}^{\text{KKQT}}(\omega) &= \frac{\omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega), \\ \Phi_{B,-}^{\text{KKQT}}(\omega) &= \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega),\end{aligned}\quad (\text{B1})$$

from Ref. [41] (KKQT), where $\bar{\Lambda} = M_B - m_b$, and we use pole mass $m_{b,\text{pole}}$ following argument of Ref. [42].

In our preferred q^2 range, modifications in the q^2 distributions $d\hat{B}/dq^2$ by the use of these light-cone distribution amplitudes are small; within 10% (1%) for the charged (neutral) B decays. These shifts are within the error

bands shown in the left panel of Fig. 2. On the other hand, as shown in the left panel of Fig. 6, the distortion of $A_{CP}^+(q^2)$ is more significant, although the overall behavior is similar to the “GN model” (Ref. [38]) used in the main text. The q^2 distribution drops sharply down to (minus) a few percent level at $q^2 = 2\bar{\Lambda}M_B \sim 5 \text{ GeV}^2$ due to the sharp $\omega = 2\bar{\Lambda}$ cutoff in Eq. (B1), in contrast to the smooth damping in GN model. The magnitude of $A_{CP}^+(q^2)$ gets enhanced (suppressed) below (above) $q^2 \sim 4 \text{ GeV}^2$ up to 30% (70%). Nevertheless, the q^2 -averaged $\langle A_{CP}^+ \rangle$ does not change from the GN model case, because of an accidental cancellation for our particular choice of $2 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ range. $A_{CP}^0(q^2)$ also gets distorted, but the shift is small, within the error band of the GN model prediction.

As shown in the right panel of Fig. 6, $A_I(q^2)$ is enhanced for q^2 below 5.4 GeV^2 . The enhancement is above 50% for $1.2 \text{ GeV}^2 \lesssim q^2 \lesssim 4.8 \text{ GeV}^2$, with the maximal enhancement $\sim 90\%$ at $q^2 \sim 3.5 \text{ GeV}^2$. The q^2 -average is also enhanced, $\langle A_I \rangle^{\text{KKQT}} = 0.05$, but is still below 10%.

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